# SPACEABILITY OF SETS IN $L^p \times L^q$ AND $C_0 \times C_0$

**FILIP STROBIN**
**(WITH SZYMON GŁĄB)**
INSTITUTE OF MATHEMATICS OF THE ŁÓDŹ UNIVERSITY OF TECHNOLOGY

Abstract. A subset $E$ of an infinitely dimensional linearly–topological space $X$ is called spaceable if there is an infinitely dimensional closed subspace $Y$ of $X$ with $Y \subset E \cup \{0\}$. During the talk I will present the spaceability of the following sets:

1. the set of those $(f, g) \in L^p \times L^q$ for which $fg \notin L^r$ provided that one of the following conditions holds:
   (a) $0 < \frac{1}{p} + \frac{1}{q} < \frac{1}{r}$ and $\sup\{\mu(A) : \mu(A) < \infty\} = \infty$
   (b) $\frac{1}{p} + \frac{1}{q} > \frac{1}{r}$ and $\inf\{\mu(A) : \mu(A) > 0\} = 0$;
2. the set of those $(f, g) \in C_0 \times C_0$ for which $fg$ is not integrable, where $C_0$ is the space of continuous mappings which vanish at infinity;
3. the set of those $(f, g) \in L^p(G) \times L^q(G)$ for which the convolution $f \star g$ is not well-defined or is infinite, provided $G$ is a locally compact non-compact topological group and $p, q > 1$ with $1/p + 1/q < 1$.

The results are related to our previous ones [GS1], [GS2], [GS3] in which we studied these sets from the Baire category and $\sigma$-porosity points of view.

[GS1] Głąb, S.; Strobin, F. Dichotomies for $L^p$ spaces. *J. Math. Anal. Appl.*, **368** (2010) 382–390.

GS2
Głąb, S.; Strobin, F. Porosity and the $L^p$-conjecture. *Arch. Math.* **95** (2010), 583–592.

GS3
Głąb, S.; Strobin, F. Dichotomies for $C_0(X)$ and $C_b(X)$ spaces, *Czechoslovak Math. J.*, **63** (138) (2013), no. 1, 91–105.